THEORETICAL AND EXPERIMENTAL INVESTIGATIONS ON MASS AND HEAT TRANSFER AT BODIES IN STEADY FLOW

BY

NILS FRÖSSLING

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ALMQVIST & WIKSELLS BOKTRYCKERI AB

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Inaugural dissertation

BY

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BY DUE PERMISSION OF THE FACULTY OF MATHEMATICS AND NATURAL SCIENCE OF THE UNIVERSITY OF LUND TO BE PUBLICLY DISCUSSED IN THE LECTURE ROOM OF THE INSTITUTE OF THEORETICAL PHYSICS AND MECHANICS, OCTOBER 6TH 1958, AT 10 A.M., FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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The work of which this pamphlet is a summary includes the following papers (in the following pages referred to as *A* ... *D*):

A. *Über die Verdunstung fallender Tropfen.*

Gerlands Beiträge zur Geophysik *52*, 170, 1938.

B. *Verdunstung, Wärmeübergang und Geschwindigkeitsverteilung bei zweidimensionaler und rotationssymmetrischer laminarer Grenzschichtströmung.*

Lunds Universitets Årsskrift. N.F. Avd. 2. Bd 36. Nr 4. 1940;
Kungl. Fysiografiska Sällskapets Handlingar. N.F. Bd 51. Nr 4.

C. *Calculation by series expansion of the aerodynamic heating at laminar constant-property boundary layers.*

Lunds Universitets Årsskrift. N.F. Avd. 2. Bd 54. Nr 9. 1958;
Kungl. Fysiografiska Sällskapets Handlingar N.F. Bd 69. Nr 9.

D. *Calculation by series expansion of the heat transfer in laminar, constant-property boundary layers at nonisothermal surfaces.*

Arkiv för Fysik *14*, 143, 1958.

The papers included in the present work are reports from a series of investigations on mass and heat transfer which the author began at the University of Lund and continued at the Royal Institute of Technology, Stockholm.

Paper A deals with the evaporation of falling drops. The author first met this problem during work on chemical warfare defence. It was also found to be of interest in meteorology and chemistry. Furthermore, evaporation and heat transfer are, to a certain extent, analogous phenomena and are both dependent on the flow around the drop. Since, for falling drops, the Reynolds numbers Re cover a region including large and small frictions, the problem is of basic interest for the study of flow and of mass and heat transfer.

When the investigation was begun, the evaporation of a drop in a fluid at rest had been treated in several works. However, very little was known about the so-called wind factor f, by which the total evaporation was multiplied when there was a relative movement between the fluid and the drop. Some calculations were earlier made by Fuchs for small Reynolds numbers. During the course of the work, some approximate experimental values by Takahasi became available. For the calculation of either the mass transfer or heat transfer, it is necessary first to obtain information concerning the velocity field around the body, and then study the distribution of mass or heat in that field and finally calculate the flux at the wall from the gradient of concentration or temperature (since the fluid is at rest at the wall). Because of the complexity of the pertinent equations, this problem cannot be solved in the general case. In the theoretical part of paper A, f was found by means of similarity analysis to be a function of Re and of the material constant $\sigma = \Delta/\nu$. According to Fuchs, f should approximately equal 1 at small Re. At such large Re values, that the boundary layer approximations could be permitted, f was found to be $k\sqrt{Re}$, where k could be expected to be approximately inversely proportional to σ^n with n about 1/3. An experimental investigation was performed. Drops of nitrobenzene, aniline and water were placed on thin glass rods or thermoelements and an air-current from a wind tunnel was blown from below. The evaporation was measured by photographing the drops repeatedly on a magnified scale. The radius of the drops was 0.1–0.9 mm, the wind velocity 0.2–7 m/sec and Re 2–800. Solid spheres of naphthalene were studied in the same way and thus the distribution of the evaporation over the surface could also be studied. Several sources of error were discussed and taken into account. All the measurements were in good agreement with the formula

$$f = 1 + \frac{0.276}{\sqrt[3]{\sigma}}\sqrt{Re} \qquad [1]$$

In the range examined the drops behaved as solids. Formula [1] has been used and confirmed in studies in meteorology (1,10), chemistry (2,14), combustion (3) and heat transfer (4). In (11), the author's experimental values and the form of the

equation [1] have been confirmed by means of measurements, in a smaller range of *Re*, on aniline and water. However, for the constant factor in the second term, the higher value 0.30 was obtained for water since a lower value had been used in (11) for the diffusion constant of water. Accurate measurements in e.g. (12) of the latter constant confirm, however, the value used by the present author. In (13), from experimental values with a rather large spread, another form for f was proposed for low *Re*.

The technique for investigating the evaporation from solid spheres has been later improved by the author and measurements have been made for *Re* up to 4.10^5. This material is still being treated and the results are as yet unpublished. The method for measuring transfer coefficients by means of the evaporation of naphthalene—used, for the first time, by the author in paper A—has become a standard method for measuring the distribution of heat transfer coefficients since the latter coefficients can be calculated from the former with a certain degree of accuracy (5).

In order to calculate the distribution of the transfer over a solid body, the author made a theoretical study (*paper B*) for that region of *Re* where a thin laminar boundary layer exists since the equations are then not as complicated as in the general case. The calculations were made for incompressible flow. For this case, the mass and heat transfer have quite analogous equations. At that time, exact solutions of the transfer problems had only been published for a flat plate and for a two-dimensional stagnation point. An approximate solution had been given for a cylinder by Kroujiline who used, for the temperature field, a distribution of the same type as that employed by Pohlhausen for the velocity field. It was, however, desirable to obtain a solution that did not use any such further assumptions besides the basic equations. Blasius and Hiemenz had shown that such a solution could be made for the two-dimensional velocity field at blunt bodies by means of a power series expansion with respect to the distance x along the wall from the stagnation point. The coefficients of the powers of x were functions only of the distance y from the wall. Howarth had shown that the coefficient functions could be divided up into terms containing universal functions. These functions did not depend on the particular data of the problem and could therefore be calculated once and for all. The series expansion, being an exact solution, could be used within its region of convergence and could be extended by some step-by-step method, e.g., that by Prandtl and Görtler. In this way a method of calculation was given, the error of which depended only on the amount of work expended during the calculations and not on any approximate assumptions. In paper B, the author used this treatment for the two-dimensional transfer layer and thus gave the first general exact solution for the transfer problem at blunt bodies. The functions occurring in the transfer boundary layer could also be divided into parts containing universal functions. For the axially symmetrical case (a more complicated case because of the form of the equation of continuity), no exact solutions had prev-

iously been given. The author therefore worked out the series expansions both for the velocity field and for the temperature and concentration field. Several universal functions were calculated numerically for the two-dimensional and the axially symmetrical cases. The author made afterwards further, as yet unpublished, calculations of such functions.

The first term in the author's series for transfer in the axially symmetrical case, which expresses the transfer at the stagnation point, is identical (also with respect to numerical values, cf. Table 5) with a theory by Sibulkin (6), published twelve years later and often referred to in connection with the transfer at a three-dimensional stagnation point at large velocities, e.g., (7).

The step-by-step procedure was discussed for the different cases. Later, more rapid methods than that of Prandtl and Görtler have been introduced by Görtler (8) and by Witting (9).

As regards the influence of the Prandtl number ν/a, the present author showed that the third root law was exactly valid for large Prandtl numbers.

At the end of paper B, some approximate calculations were discussed. Since then, many approximate methods have been described in the literature.

By request of the National Advisory Committee for Aeronautics, U.S.A., paper B has been translated into English and published as a NACA TM 1432, 1958.

In *paper C* some calculations concerning aerodynamic heating are described. When a fluid flows past a rigid body, heat will be generated in the fluid due both to internal friction (dissipation) in the boundary layer and to pressure changes. This phenomenon becomes more intense with increasing velocity. Owing to the unevenness of this heating, a poorly conducting body will be subject to a temperature variation over its surface. At equilibrium (after correction for possible radiation) there is no heat transfer from any point of the surface of the body, and the corresponding local value of the wall temperature is called the adiabatic wall temperature T_{aw}. It is of great importance to be able to calculate the equilibrium temperature field and especially the T_{aw} at different points on the surface of a nonconducting body. However, even if the body is noninsulating these values will still be of interest. This is due to the fact that the heat transfer between a body and a fluid is determined by the differences between the surface temperature and this T_{aw} at different points (at least at not too high Mach numbers and not too large temperature differences). A considerable amount of work had earlier been done theoretically and experimentally on adiabatic wall temperatures. Eckert and Drewitz gave reasons for assuming the changes of pressure and temperature to be so small for bodies of arbitrary shape that it was possible to use constant properties of the fluid, at least for subsonic speeds. This simplified the calculations a great deal. Since then several exact solutions have been given for constant and variable properties at flat plates, wedges and cones. Constant-property solutions can sometimes be useful even for supersonic velocities. One

example of this is the flow in the neighbourhood of the stagnation point of a blunt body, since there will then be a detached shockwave and, behind it, a subsonic region of a certain extension.

For calculating boundary layers at bodies of arbitrary shape, there are available several approximate methods which, however, require a further assumption together with the fundamental equations. It therefore was of interest to study a constant-property treatment of this problem, since the author found that an exact solution could be obtained in that case by using a series expansion of the same type as that used in paper B. In paper C this solution is worked out and some of the pertinent universal functions are calculated numerically.

As one result of the calculations the local recovery factor was, in agreement with experiments, found to be constant over the surface, at least as far as the terms included in the calculation show.

In paper B the temperature (or the concentration) was supposed to be the same at all points on the surface of the body. In *paper D* the author extended that theory to the case when the temperature (or concentration) had a power series distribution over the surface. The latter problem could arise at small velocities with negligible viscous heating or at larger velocities. The author showed that the solution of this more general problem could be obtained by superposing solutions of special cases of those in paper B, with some of the functions being given new boundary values. Some of the functions occurring in this problem were calculated numerically.

The author wishes to express his sincere gratitude to Professor T. Gustafson, Director of the Institute of Theoretical Physics in Lund, for his kind and encouraging interest in these investigations and for many interesting discussions, especially during the first part of this work, when the author lived in or often visited Lund.

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